Invited Lecture

Challenging Tasks: Opportunities for Learning

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ABSTRACT Challenging tasks are essential in developing and demonstrating mathematical understanding. They provide opportunities to learn and the motivation for student to engage with learning. This chapter highlights how the real-world and digital technologies provide many opportunities to design and implement challenging tasks for all learners. The affordances of technology-rich teaching and learning environments need more attention if teachers and their students are to be better enabled to maximise opportunities for learning mathematics. A range of tasks are presented and discussed. Planning by teachers for varied student responses is critical in enabling 'as needed' in-the-moment scaffolding to keep students engaged with mathematical thinking.

Keywords: Affordances; Challenging tasks; Cognitive demand; Digital technologies; Engagement; Real-world.

1. Challenge

Jaworski (1992) described mathematical challenge as part of what is required, for students, to learn mathematics. She argued that mathematical challenge can only be realised where attention also is given to the supportive learning environment that fosters learning. Challenge "involves stimulating mathematical thought and enquiry, and motivating students to become engaged in mathematics thinking" (p. 8). It influences task design and implementation and the environment where learning occurs. Opportunities to engage in mathematical thinking, "cannot be taken up if it is inappropriate, or if strategies for handling it have not been created … but challenge is required to get mathematics done" (p. 14).

A focus on the learning environment was explored by Wood (2002) and others with a particular focus on how the expectations correlated with the level of mathematical thinking. Wood showed how elements of a classroom culture that fostered the development of mathematical thinking included shifting the responsibility for thinking and participation in discussion from the teacher to the students. Critically, she describes the need to enable students to become active listeners and explainers. This notion that the focus on mathematical thinking in the classroom should be undertaken by learners, facilitated, and scaffolded by the teacher as needed, with the mathematical thinking of the teacher occurring beforehand (i.e., during planning), has been well explored by Smith and Stein and colleagues (e.g., Smith and Stein, 2011,

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Stein, Engle, Smith, and Hughes, 2008). These expectations, of learners in class, and teachers as part of planning, are critical to challenge, and hence mathematical learning (Jaworski, 1992) being realised. The importance of the teacher's role is further detailed by Baxter and Williams' (2010) discussion of discourse-oriented teaching.

2. Challenging Tasks

A *challenging* task is, by definition, high in cognitive demand. However, what teachers say or do as students begin tasks, typically reduces the cognitive demand faced by students (González and Eli, 2017; Smith and Stein, 2011). Interactions between the teacher and student, particularly, but not limited to, the beginning stages of task solving can be planned for or involve in-the-moment decisions. Challenging tasks should engage students in their learning of mathematics. Motivation to learn and to value mathematics can be influenced by the task itself and how both teacher and students interact with the task. Participation by countries in PISA suggests it is a clear expectation that students do engage successfully with challenging tasks.

Several frameworks have been developed to consider the level of challenge of tasks and their implementation. Stillman (2001) developed a cognitive demand profile to analyse applications tasks, particularly those intended for upper secondary mathematics students. She notes, "the cognitive demand is related to the interaction between the mathematical demand of the task and the extent to which the mathematics needed to model the situation is embedded in its description" (p. 459). Stillman, Edwards and Brown (2004) extended this tool to consider the task implementation, that is, what the teacher and students 'do' with the intended task. They developed a framework "for engineering the cognitive demand of tasks, lessons and lesson sequences" (p. 489) that considered three mediators of cognitive demand, namely, task scaffolding, task complexity, and complexity of technology use. They found that by "orchestrating the interplay between degree of task scaffolding, task complexity, and complexity of technology use, teachers are able to craft lesson sequences involving tasks of appropriate level of challenge for their students" (p. 500).

Challenging tasks can be of a variety of types. They include problem solving tasks of varying length, mathematical modelling where the focus is on solving a real-world problem, and investigative tasks. They involve 'doing mathematics', as distinct from memorisation or procedures without connections, and are of high cognitive demand (Smith and Stein, 2011). Barbeau describes a challenge as, "a question posed deliberately to entice its recipient to attempt a resolution, while at the same time stretching their understanding and knowledge" (2009, p. 5). Noting that challenge depends on background and interest, he argues that good challenges involve explanation, questioning, multiple possible approaches, and evaluation of the solution.

Following Francisco and Maher (2005), learners often develop rich understandings of key ideas through solving complex, challenging tasks, particularly when given the opportunity to explore inter-related tasks. For example, students learn to model as they engage with multiple modelling tasks over time. Students need time to develop modelling competencies through modelling, in the same ways students "need time to develop and come to understand the importance of a way of working based on sense making and justification of ideas" (2005, p. 368).

Challenging tasks are critical learning activities at all levels of schooling. Without mathematically challenging tasks being a normal part of the teaching and learning environment, opportunities to learn are limited. Real-world tasks are by their very nature challenging. By real-world tasks, I refer to task solvers solving *real problems*. This includes making sense of the real-world context, making decisions about what is relevant and important, and mathematising the problem - bringing the problem into the mathematical world so that it can be solved. Once solved, the mathematical solution must be interpreted in terms of the real world to ascertain what the real-world solution is and if this is acceptable. Typically, in mathematical modelling, the initial real-world solution would be explored further, perhaps by relaxing some of the simplifying assumptions, accounting for additional factors, or revisiting estimates of important factors, to seek an improved real-world solution. At the very least, students should reflect on their solution and consider varied assumptions or estimates or approaches.

Teachers play a critical role in providing students with challenging tasks, and also ensuring the cognitive demand remains high during task implementation. A challenge for many teachers is to maximise the mathematical thinking done by the students, rather than themselves during the lesson. A technology-rich teaching and learning environment also poses a challenge for teachers. Affordances of such an environment must be both perceived and enacted by students during task. Prior to this occurring, teachers must provide opportunities for students to experience such affordances and consider their applicability or usefulness in different mathematical situations.

2.1. Issues implementing challenging tasks

According to Sullivan et al. (2015), teachers typically view mathematics as procedural (p. 124). This view of mathematics as a set of disconnected skills, results in the belief that challenging tasks have no place in the teaching and learning of mathematics. Furthermore, teachers consider students "reluctant to engage with challenge … and unwilling to persist" (p. 124) when faced with challenge. Furthermore, teachers typically reduce the cognitive demand of tasks when planning. Earlier, Stein et al. (2008) found that teachers tend to overexplain tasks when implementing them, thus reducing the cognitive demand for students as some of the mathematical thinking is undertaken by the teacher rather than students. Russo and Hopkins (2019) identified "issues related to teachers' self-perceived capacity to teach with such tasks" (p. 760).

Consideration is also needed when it comes to student engagement with challenging tasks, when they do experience them. Williams (2014) makes an important distinction between persistence and perseverance. She describes perseverance as occurring when task solvers, find ways to proceed toward successes when situations are unfamiliar, and a clear pathway is not apparent. Williams (2014) found that elements of perseverance underpin creative problem-solving. In contrast, persistence is when the task solver keeps on trying, no matter the quality of the attempt, when difficulties are encountered (p. 420). Recognising this difference will enable teachers and students to consider alternative pathways when blockages (Stillman, Galbraith, Brown, and Edwards, 2007) or dead ends are reached. Furthermore, anticipating solution pathways (e.g., Stillman and Brown, 2014) might also see task solvers more likely to persevere rather than simply persist, or give up. Teachers might shift their focus from encouragement to prompting about considering alternative pathways.

3. Factors Impacting on the Level of Challenge

In this section three factors that impact on the level and extent of challenge possible, and being realized, are discussed. Initially each of these, the real-world, affordances and digital technologies, are carefully defined. This is critical as, in particular the first two are used in multiple ways in the literature and too often without definition. Interactions between the factors are then discussed.

3.1. Real-world

Mathematical modelling tasks in school involve a genuine link with the real-world. The real-world is critical at the beginning and end of the task. It is also necessary to keep in mind throughout the solution process although at times to a lesser extent than others. This is referred to by Stillman (1998) as context as tapestry rather than context as wrapper or context as border. In the latter two categories the real-world can be thrown away after initial consideration or ignored altogether as it is superficial to the problem. (See also Brown, 2019.) This is not the case in genuine modelling tasks.

Solving modelling tasks necessitates some level of complexity of mathematical thinking, that is, higher order thinking, by task solvers (Brown and Edwards, 2011). Higher order thinking is "taken to mean instances where there is evidence that a student appropriately *makes choices* about the solution path; … makes links across representations; expects to verify a conjectured solution; appreciates the value of, or need for verification" (Brown and Edwards, 2011, p. 190). Resnick (1987) noted that whilst defining higher order thinking may be problematic, recognising it is not.

3.2. Affordances

Following Gibson (1979) who invented the term, *affordance*s are opportunities for interactivity between actors and their environment expressed in a linguistic form (i.e., < … > -ability) to indicate this *opportunity*. Brophy (2008) describes the affordances insightfulness-ability, understand-ability, information processing-ability, problem-solving-ability and decision-making-ability as important in increasing motivation to learn and valuing of mathematics as a discipline by students.

Some see affordances as a property of an object, but this is not the interpretation taken here. Following Gibson (1979), affordances are part of an actor-environment system. "If an affordance exists, in order to avail oneself of the opportunity for interactivity, the actor must act (on or with the object). The precursor to acting is *perceiving* — without which the actor cannot act when the affordance is for teaching or learning" (Brown and Stillman 2014, p. 112).

3.3. *Digital technologies*

Access to digital technologies can change the complexity of tasks student explore. More complex or larger and hence more realistic data sets can be explored. Calculations beyond one's by-hand capabilities are accessible. Graphing calculator or equivalent computer-based technologies allow for the making and testing of conjectures, and exploring ideas, that would not otherwise be possible. CAS-enabled technologies provide upper secondary students the capabilities to apply calculus related ideas to any functions. Digital technology use provides opportunities for students to deepen and expand their mathematical knowledge. In many educational jurisdictions, digital tools are available and expected to be used for teaching and learning mathematics. It does not follow necessarily that challenging tasks become the norm. In other jurisdictions, for various reasons, digital tools are less accessible or absent.

Often when opportunities exist for teachers to deepen understanding or use more complex problems, afforded by the presence of available digital tools, they often do not do so even when possessing the relevant knowledge (Brown 2013b, 2015a, b, 2017). Similarly, students are reluctant to use their knowledge of the available digital tools to expand or deepen or demonstrate their mathematical understanding. Teacher expressed intentions to allow students to make decisions with respect to task interpretation and understanding and choice of digital tools and the ways they are used is often not realised in practice (Brown, 2017). Knowledge of teacher tactics and expected responses (Brown, 2013b) and potential student strategies (Brown, 2015a, b) play a critical role in teacher preparedness for successful task implementation where the mathematical thinking is predominantly undertaken by the students.

3.4. *An Illustration*

Consider the *Platypus Task*, which is briefly described here. This task was implemented where digital tools, namely graphing calculators were expected to be used. The *Platypus Task* was solved by Year 11 students (aged 15–16 years). Their school was located close to the Yarra River where platypus have lived for many millenia. The platypus, an Australian monotreme (i.e., an egg laying mammal) is in danger of local extinction. The task context was used to either already be of interest to students or to encourage an interest in their local environment.

The task presented students with 'local area' data before and after an intervention project. Two questions posed were: What was predicted for the platypus 'local' population before intervention? What is the situation post intervention? The graphing calculators used by the students allowed multiple affordances to be perceived and enacted in successful solution of the task.

The selected analysis of one student's engagement with this challenging task illustrates that affordances of the technology-rich environment need to be both perceived and enacted. The affordance of interest here is *multiple function view-ability*. Enactment of this affordances allows the task solver to view their plot and function model for the two data sets on the same set of axes. This is critical in order to easily see the effect of the intervention. In this case, the student, Sali, did not perceive, nor enact, the available affordance during task solving.

In her post-task interview, Sali suggested it was *not possible to compare* the *before* and *after* models. Looking at her by-hand sketches, she said,

> Well, they both have different scales so you can't really tell which one. [pause] Obviously, this one looks like it is more steadily decreasing than the other one but I wouldn't be able to tell you which model is exactly better than the other one.

When asked why she had used only two lists (in her calculator she had deleted the 'before intervention' data, in order to consider the 'after intervention' data), it immediately occurred to her that four lists would have allowed her to see all the data together and compared the models for each data set as required. Note the default setting for the graphing calculator used was six lists and more could be added.

> *Sali:* Oh, no, I should have done that because then you could have seen them both together then you could have put the line in … I should have done that. …

> *Sali:* Yeah, you would look at it and they would be on the same, roughly the same scale so you would be able to have a more accurate look at them and see which one was decreasing faster or leveling out or whatever. Awh!!

Consider the difference in opportunities to analyse the effect of the intervention shown by Sali's views of the data and models (viewed sequentially, different data set) (see Fig. 1) with those of Chris — all data and two models visible simultaneously. Differing viewing domains and ranges, added to the difficulty of analysis.

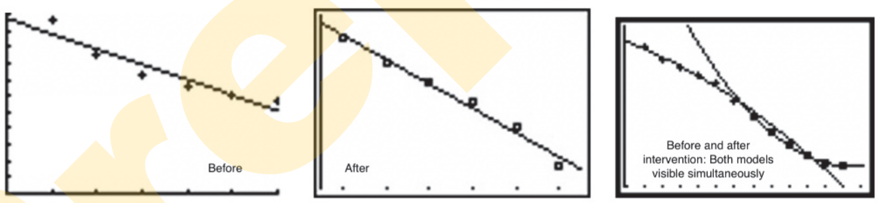


Fig. 1. Sali's (a) pre- and (b) post-intervention data and model view, (c) Chris's simultaneous views

3.5. Perceiving and enacting affordances in a TRTLE

In unpacking the complexity of the task and its solution in a technology-rich teaching and learning environment, several factors need to be considered as students perceive and enact affordances. The primary issue for teachers as observers of classroom learning experiences on which to base judgements about enactment of affordances of technology-rich teaching and learning environments (TRTLEs) (e.g., Brown, 2015a) in function situations is how students manage their competence in enacting affordances as they attempt to solve a function task. Fig. 2 shows that in students' perceiving and enacting affordances of a TRTLE to solve function tasks, both mathematical content knowledge (of functions) and technological knowledge (of digital tools) are required by the teacher. In addition, both perception and enactment of affordances by students need to occur if independent student use of the technology in solving functions tasks is to occur. In a TRTLE, the affordance bearer is broadly speaking some feature of the digital technology (e.g., Window, Zoom) whereas the affordance is the interactivity between the user for some particular purpose (Function View-ability).

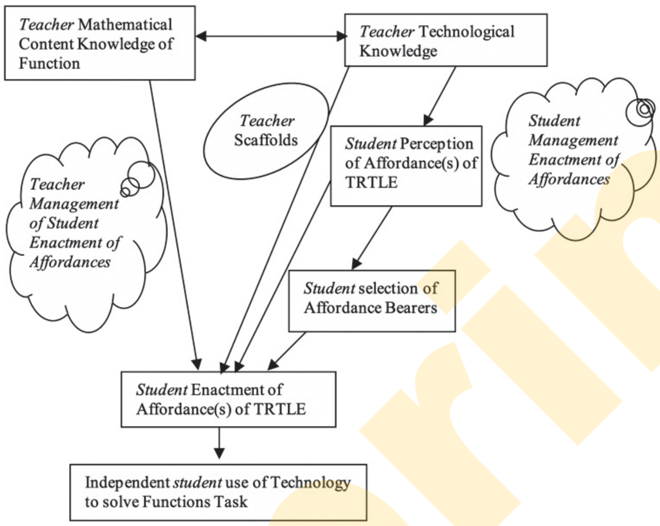


Fig. 2. Factors involved in enabling student independent technology use

Knowledge of teacher tactics and expected responses (Brown, 2013b) and potential student strategies (Brown, 2015a, b) play a critical role in teacher preparedness for successful task implementation where the mathematical thinking is predominantly undertaken by the students. Teacher management of students' engagement with challenging tasks is complex. See also Baxter and Williams (2010). This is even more so in a technology-rich teaching and learning environment.

4. Modelling, Mathematics Content, and Digital Technologies

Galbraith, Stillman, Brown, and Edwards (2007), researchers and a teacher in the RITEMaths project, looked at the interactions between modelling, mathematics content, and technology. A similar approach is taken here. Fig. 3 illustrates the possible interactions between two or three of mathematical modelling (MM), digital technology (DT), and mathematics content (MC). Interactions can occur between two or three of these. In brief, interactions include *Interactions between* MM *and* DT: MM ∩ DT (Strategy decisions. Choice of technology, selection of a particular technological function); *Interactions between* MM *and* MC: MM ∩ MC (How is the problem represented? What is the goal?); *Interactions between* MC *and* DT: MC ∩ DT (Use of calculations, plots, graphs, approach to verifications, varying specific case to more general); and *Interactions between* MM *and* DT *and* MC: MM ∩ MC ∩ DT: (Carrying through to solution: formulation … approach, use of representations, interpreting results (algebraic, numerical, graphical) → interpretation → implications).

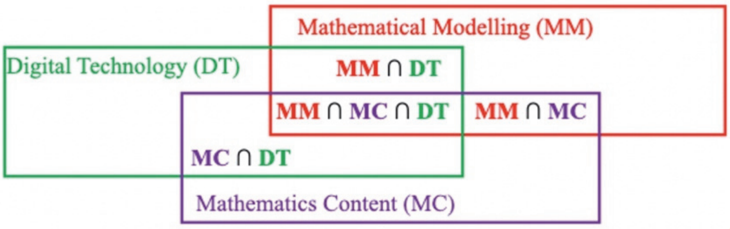


Fig. 3. Interactions between mathematical modelling, mathematics content, and digital technology

4.1. Interactions between Teacher, Students, and Learning Activities.

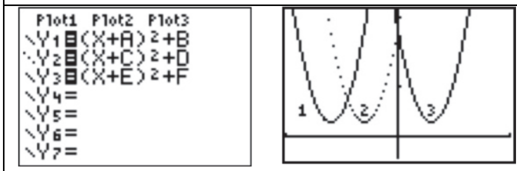
In addition, we can also consider interactions between teacher, students and learning activities. Work by the Learning Mathematics for Teaching Project (LMTP, 2011) team, which included Hill, Ball, and Bass, makes it clear a focus on *teachers, students, content,* and the *interactions* among these is essential for quality teaching and learning. The teacher is responsible for determining lesson content whether this be a task or learning activity, albeit influenced by the intended curriculum. During class, teachers manage the dynamic interactions including what they say and do, and "what students say and do, and what a curriculum affords" (p. 30). A task may have potential, but if students do not engage with it, this potential is not realised. Similarly, a highly motivating task that usually engages students may miss the mark if the teacher fails to notice teachable moments and focuses on trivial or irrelevant mathematical features.

4.2. Digital technology: Opportunities for improved task design MC ∩ DT

Consider the two tasks shown. These tasks were written by the author in the late 1990s. The digital technology here is the TI-83 graphing calculator. These tasks are part of a larger set of tasks designed for two classes of Year 11 students in a curriculum context where graphing calculator use was mandated. Student were expanding their knowledge from simple polynomials to include power and transcendental functions and beginning to learn calculus where algebraic, graphical and numerical representations play an important role. Irrespective of whether the technology was available during task solving, the TRTLE provided enhanced opportunities to develop and demonstrate conceptual understanding. These tasks illustrate interactions between mathematical content and digital technologies. What is possible depends on the technology available, the teacher, and the students. In this circumstance, the availability of DT allowed the teacher-researcher to rethink her teaching approach. Task design provided opportunities to (a) focus on generality, (b) consider functions using a multiple-representation approach, and (c) explore, conjecture, test, and check. Although the tasks presented were originally used to assess student understanding of functions, they can be used as learning activities. It was an important point in time where the teacher-researcher saw ways to make learning accessible to more students.

Example Task 1

Given the equations and graphs below what can you say about the values of A, B, C, D, E and F? Can you suggest a possible set of values for A, B, C, D, E, and F?



Example Task 2

The following table shows some values of $y_1(x)$ and $y_2(x)$.

X	Y1	Y2
0	0	1
1	1	0
2	4	1
3	9	4
4	16	9
5	25	16
6	36	25

Write down the equation of $y_2(x)$ in terms of $y_1(x)$.
Explain how you did this.

4.2.1. Multiple representations: algebraic, graphical, and numerical

The tasks were specifically designed to provide opportunities for students to consider translations *within* and *between* representations (See Fig. 4). Note the bi-directional arrows, drawing attention to translations, for example *from* the graphical *to* the algebraic representation and *from* the algebraic *to* the graphical representation. The single headed arrows indicate translations within that representation, for example considering two different graphical representations of the same function.

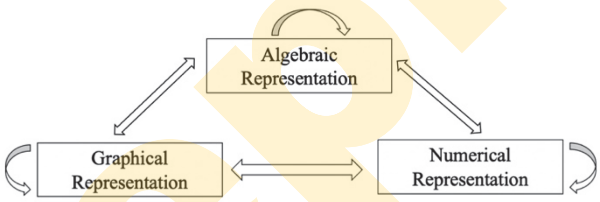


Fig. 4. Translations within and between representations

Kaput (1989) suggests that multiple linked representations allow learners to combine understanding from different representations and build a better understanding of complex ideas and apply these ideas and concepts more effectively. In secondary school mathematics curricula, specifically focusing on functions Romberg, Fennema and Carpenter (1993) drew our attention to the notion of multiple representations of functions, the importance translations among representations, and three critical representations, referred to here as algebraic, graphical, and numerical. The numerical representation appears to still be undervalued (Bannister, 2014).

Whilst Example Task 2 appears to focus the task solver's attention on the numerical and algebraic representations, opportunities exist to consider the graphical representation. The numerical representation provided shows that $y_2(x)$ has the same value as y_1 for the previous x value (i.e., when x is 1 less). This can be considered as graphically y_2 is the same shape as y_1 but shifted 1 unit right. Zazkis, Liljedahl, and Gadowsky (2003) have shown that this transformation is typically less well understood than others — by teachers and students, and still is a current focus in mathematics education research (Sudihartinih and Purniati, 2018).

The collection of tasks, from which these two are selected, gave equal value to each representation, and focused student attention on generality. A study by Brown (2003), including the collection of items found more attention was given, by students, to across, rather than within, representation notions, albeit there are more possibilities for the former. Not only was greater attention given to across, rather than within, representation movement, but also there was not and even spread across representational pairs. Perhaps not surprisingly, connections between representations, where the initial representation was the numerical, received the least attention.

4.3. Real-World Problems and Primary Students MC ∩ MM

In this section, we consider challenging tasks with a focus on the real-word and mathematical content. Three tasks (*Letters, Brass Numerals, Packing Truck*), from two research projects[2] are briefly presented and their learnings from their implementation discussed. All three tasks are research designed, one by the author, one co-designed with a colleague, and the third modified from Swan (2015).

Mathematical modelling and applications provide opportunities for teachers to teach in engaging ways and students to become increasingly confident in working with challenging mathematical tasks. Yet their use remains less common in the primary years of schooling. Both research projects had a focus on improving the quality of teaching and learning. Design and implementation of the task formed part of that approach. Teacher observations provided opportunities for them to see what level of challenge their students engage with.

4.3.1. The Letters and Brass Numerals tasks

The *Letters Task* involved preparing and communicating advice to a local toy store owner who was considering a new product. The items of interest were painted wooden letters and can be used to decorate a bedroom door. Harvey might want only his initial H, whereas Darcy might prefer 5 letters to display his full name. How many of each letter should the bookstore owner order, with reasons, so future orders can follow the same strategy (Brown, 2013a). Although students engaged with the *Letters Task*, few interacted with the real-word context. Many groups saw the task as a (challenging) division problem. Most groups made no use of data, although easily accessible. Teacher — student interactions may have directed focus away from the real-world and to the mathematical world. Had students felt more comfortable using, a four-function calculator — they may have considered the context more important.

The *Brass Numerals Task* involved determining how many brass numerals the local chain hardware store (Bunnings) should have in stock, to satisfy customer needs for identifying their house (12) or unit number (2|302) so mail was delivered to the

[2] CTLM: Contemporary Teaching and Learning Mathematics Project — funded by the Catholic Education Office Melbourne. TALR: Teacher as Learner Research Project (unfunded).

correct location. (See Brown, 2013a, Brown and Stillman, 2017). This task — perhaps similar at a surface level, saw increased engagement with the challenging real-world problem. Task design meant students could no longer throw away the real-world aspects and treat the context as wrapper (Stillman, 1998). In-the-moment the teacher thought students needed more time and an extra lesson was allocated for students to continue working on the task. This additional time enabled some students to take notice of numbering in their neighbourhood and increased attention to the frequency of each digit. This new contextual knowledge was shared with their group the next day.

Implementing these two different tasks, highlights the importance of task design impacting on whether students saw the task as realistic (Brown 2013a). It is clear that, "tasks that required students to reflect … and make their thinking explicit can contribute to … students perceiving themselves as playing an important role in interpreting the real-world problem situation and relating it to the world of mathematics" (p. 304). The requirement to communicate the solution to an outsider (e.g., toy store owner, ordering manager) had proved particularly helpful in scaffolding students to clearly communicate their solution.

4.3.2. Packing truck task

The previous two tasks were implemented with Grade 6 students (aged 10−11), whereas the *Packing Truck Task* (Brown, 2021, Swan, 2015) was for youngers grade 3–4 students (aged 7−9 years old). In-task scaffolding was included for these young novice modellers. The task began by considering packing inside a box, then shifted to stacking these boxes. Finally, the task considered how many of these boxes could be packed in a truck (dimensions specified). To successfully solve real-world tasks, students must notice what is relevant, and decide how to act on this to progress their solution. Teachers must also discern what is relevant and nurture student capacity to notice. What teachers do matters and is critically important. In the *Packing Truck Task* most students attended to real-world aspects of the task. However, *little evidence* was found of teachers attending to this as they observed. Improved teacher noticing of real-world considerations and hence asking questions of students when stuck, or off-track, that better support student use of realistic considerations would have increased student success with the task. Teachers need to focus attention on explaining reasoning — not why a particular calculation gave a particular result, but where the numbers come from — what they represent and the operation and/or result making sense? Students need more experiences with such tasks as argued by Francisco and Maher (2005) and others.

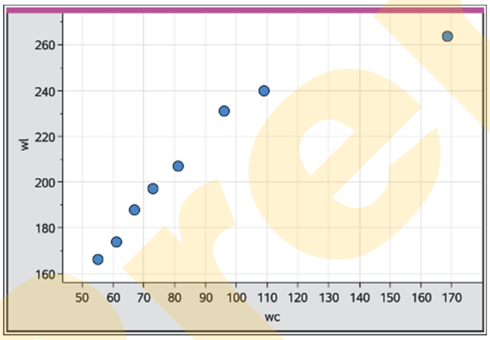
4.4. Quality teaching

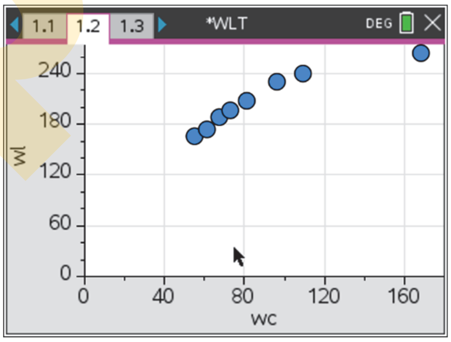
For teachers and researchers, it is *a challenge to design or select* a task where students will engage with the real world. Task design depends, in part, on typical approaches to teaching and learning. Blum (2015) suggests the continued overuse of "dressed-up word problems" (p. 83), rather than genuine applications or modelling, is because teaching mathematical modelling and applications is demanding and related to teacher quality. He argues that high quality teaching is particularly necessary for effective teaching of mathematical modelling and applications. Recommendations from Blum (pp. 83-86) include: effective student-centred classroom environments, group work; activating learners cognitively and meta-cognitively; using a broad range of contexts; teacher encouragement of solutions different to their own; a systemic approach involving regular and long-term use of modelling; assessment valuing modelling; and beliefs and attitudes that value modelling (acknowledging these take time to develop).

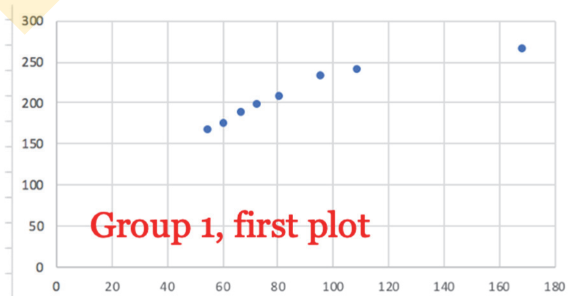
4.5. Real-world problems MC ∩ DT ∩ MC

Stillman and Brown (2021) revisited a task from Riede (2003) with Year 10 students in the Victorian component of the Enablers Research Project[3] Weightlifting is a particularly interesting sport. When the body weight categories are revised, as occurred in July 2018, all world records are nullified. The International Weightlifting Federation then needs a model to establish *reasonable minimum lifts* that can be considered as a record in each body weight category. This led us to develop the *Weightlifting Task* for students to consider in the lead up to the planned 2020 Tokyo Olympics.

The *affordance: data plot-ability* was important here to allow student to visualise the data. The plot affords task solvers greater insight into the relationship between the variables weight category (WC) and weight lifted (WL). This also brings up the importance of teachers being familiar with the digital technology. In this task, with TI-Nspire calculators used by most students, 'missing data' creates no issues (Screenshot 1). The data are simply ignored as shown. For a spreadsheet used by some







[3] Australian Research Council's *Discovery Projects* funding scheme (DP17010555).

students, this was problematic, resulting in error messages. This in turn impacted on whether, or not, students viewed that data as relevant or not. Digital technology that automatically sets a viewing window such that the Viewing domain and Viewing range are inclusive of all data can help and hinder. Some students who decided a linear model was needed, viewed the 'outlier' as something that could be removed from further consideration.

During task implementation, it emerged that some students applied different mathematical 'standards' to non-linear functions. They seemed able to ignore parts of the function beyond the domain of the data (i.e., related to extrapolation) when the function was linear, but not otherwise. This clearly impacted on their choice of model(s). Extrapolating 'to the right' was prioritised over 'to the left'. More attention was given to behaviour of the model for larger values and in contrast, little or no attention given to behaviour of the model for small values of the domain. Furthermore, some students incorrectly 'believed' a model must include the origin.

Other differences arose where different function choices led to different affordances being available. For example, if and only if, considering a linear model using the TI-Nspire, the *affordance: line move-ability* could be perceived and enacted. This allows the user to directly manipulate the graphical representation of a straight line model so as to 'best fit' the data. Many digital technologies allow enactment of the *affordance: regression calculate-ability* thus allowing users to select from a given set of function types to produce a line that statistically – not necessarily realistically - 'best fits' the data. There is no doubt the regression capabilities [often blackbox] increase the student 'toolbox' but this is often at the expense of real-world and mathematical considerations. There is no need for this to be the case as teachers could explain the process of regression, irrespective of where this sits in the local curriculum. This would allow students to be better able to decide when to use this and, if used, how the output might be interpreted. As with the Platypus Task described earlier, the *affordance: multiple function view-ability* enables students to visually compare models under consideration. From a modelling perspective, there is a big difference between deciding a quadratic model is appropriate and using regression to identify such a model and finding two regression models and selecting between them on the basis of fit only (by–eye or available statistics). Students (and some teachers) do not understand nor value this distinction.

5. Concluding Thoughts

It is well known and evidenced by mathematics education researchers that challenging tasks support mathematical understanding. We know students enjoy engaging with challenging tasks, they learn and use mathematics in doing so, and increase their valuing of mathematics and motivation to learn. The challenge for mathematics educators and researchers continues. How can we increase the number of students given substantial opportunities during schooling to experience, and being enabled to be successful, with challenging tasks? We need more students to echo, the thoughts of

Tabitha when asked: What do you think of tasks such as the Platypus Task, compared to other tasks you have done?

> I actually found it enjoyable to do this kind of thing. It is challenging and it puts to work the ability to decide where [pause] like you have got so many mathematical tools at your disposal and to be able to find out how you can apply them and how to know when to use them and that kind of thing.

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References

V. R. P. Bannister (2014). Flexible conceptions of perspectives and representations: an examination of pre-service mathematics teachers' knowledge. *IJEMST* 2(3), 223–233.

E. Barbeau (2009). Introduction. In: E. Barbeau and P. Taylor (eds.), *Challenging Mathematics in and Beyond the Classroom* (pp. 1–9). Springer.

J. A. Baxter and S. Williams (2010). Social and analytic scaffolding in middle school mathematics: Managing the dilemma of telling. *JMTE* 13(1), 7–26.

W. Blum (2015). Quality teaching of mathematical modelling: What do we know, what can we do? In *Proceedings of ICME-12* (pp. 73–96). Springer, Cham.

J. Brophy (2008). Developing students' appreciation for what is taught in school. *Educational Psychologist*, 43(3), 132–141.

J. P. Brown (2021). An opportunity for noticing by students and teachers, In: F. Leung, G.A. Stillman, G. Kaiser and K.L. Wong (eds) *Mathematical Modelling Education in East and West.* (pp. 319–330). Springer.

J. P. Brown (2019). Real-world task *context:* Meanings and roles. In: G. Stillman and J. Brown (eds), *Lines of Inquiry in Mathematical Modelling Research in Education*. (pp. 53–81). Springer. https://doi.org/10.1007/978-3-030-14931-4_4

J. P. Brown (2017). Teachers' perspectives of changes in their practice during a technology in mathematics education research project. *TTE* 64, 52–65.

J. Brown (2015a). Complexities of digital technology use and the teaching and learning of function. *Computers & Education,* 87, 112–122.

J. P. Brown (2015b). Visualisation tactics for solving real world tasks. In: G. A. Stillman, W. Blum and M. S. Biembengut (eds.), *Mathematical Modelling in Education Research and Practice* (pp. 431–442). Springer.

J. P. Brown (2013a). Inducting year 6 students into "a culture of mathematising as a practice" In: G. A. Stillman et al. (eds.), *Teaching Mathematical Modelling: Connecting to Research and Practice* (pp. 295–305). Dordrecht: Springer.

J. Brown (2013b). Teaching roles in technology-rich teaching and learning environments (TRTLE's). In: V. Steinle, L. Ball and C. Bardini (eds.), *MERGA*36 (pp. 106–113). MERGA, Melbourne.

J. P. Brown (2003). An insight into student understanding of functions in a graphing calculator environment, MEd thesis, University of Melbourne, Australia.

J. P. Brown and I. Edwards (2011). Modelling tasks: Insights into mathematical understanding. In: G. Kaiser, W. Blum, R. Borromeo Ferri and G. Stillman (eds.), *Trends in Teaching and Learning of Mathematical Modelling* (pp. 187–197). Springer.

J. P. Brown and G. A. Stillman (2017). Developing the roots of modelling conceptions: Modelling is the life of the world. *IJMEST* 48(3), 353–373.

J. Brown and G. Stillman (2014). Affordances: Ten years on. In: J. Anderson, M. Cavanagh and A. Prescott (eds.), *MERGA37* (pp. 111–118.). MERGA, Sydney.

J. Francisco and C. Maher (2005). Conditions for promoting reasoning in problem solving: Insights from a longitudinal study. *JMB,* 24, 361–372.

P. Galbraith, G. Stillman, J. Brown and I. Edwards (2007). Facilitating middle secondary modelling competencies. In: C. Haines, P. Galbraith, W. Blum and S. Khan (eds.), *Mathematical Modelling: Education, Engineering and Economics* (pp. 130–140). Horwood.

J. J. Gibson (1979). An Ecological Approach to Visual Perception. Houghton Mifflin.

G. González and J. Eli (2017). Prospective and in-service teachers' perspectives about launching a problem. *JMTE* 20*, 159-201.*

B. Jaworski (1992). Mathematics teaching: What is it? *FLM* 12(1), 8–14.

J. J. Kaput (1989). Linking representations in the symbol systems of algebra. In: S. Wagner and C. Kieran (eds.), *Research Issues in the Learning and Teaching of Algebra* (pp. 167–194). NCTM.

Learning Mathematics for Teaching Project. (2011). Measuring the mathematical quality of instruction. *JMTE* 14, 25–47.

L. B. Resnick (1987). *Education and learning to think.* National Academy Press.

A. Riede (2003). Two modelling topics in teacher education and training. In: Q.-X. Ye, W. Blum, S. K. Houston, and Q.-Y. Jiang (eds.), *Mathematical Modelling in Education and Culture* (pp. 209–222). Horwood.

T. A. Romberg, E. Fennema and T. P. Carpenter (eds.) (1993) *Integrating Research on the Graphical Representations of Functions*. Lawrence Erlbaum.

J. Russo and S. Hopkins (2019). Teachers' perceptions of students when observing lessons involving challenging tasks. *IJSME* 17, 759–779.

M. S. Smith and M. K. Stein (2011). Five Practices for Orchestrating Productive Mathematics Discussions. NCTM.

M. K. Stein, R.A. Engle, M. S. Smith and E. K. Hughes (2008). Orchestrating productive mathematical discussions. *MTL* 10(4), 313–340.

G. Stillman (1998). The emperor's new clothes? Teaching and assessment of mathematical applications at the senior level. In: P. Galbraith, W. Blum, G. Booker and D. Huntley (eds.), *Mathematical Modelling: Teaching and Assessment in a Technology-Rich World* (pp. 243–253). Horwood.

G. Stillman (2001). Development of a new research tool: The cognitive demand profile. In: J. Bobis, B. Perry and M. Mitchelmore (eds.), *MERGA24* (Vol. 2, pp. 459–467). MERGA, Sydney.

G. Stillman and J. Brown (2014). Evidence of implemented anticipation in mathematising by beginning modellers. *MERJ* 26(4), 763–789.

G. A. Stillman and J. P. Brown (2021). Modeling the phenomenon versus modeling the data set, *MTL*. https://doi.org/10.1080/10986065.2021.2013144

G. Stillman, I. Edwards, and J. Brown (2004). Mediating the cognitive demand of lessons in real-world settings. In: B. Tadich, S Tobias, C. Brew, B. Beatty and P. Sullivan (eds.), *MAV41* (pp. 489–500). MAV, Melbourne.

G. A. Stillman, P Galbraith, J. Brown, and I. Edwards (2007). A framework for success in implementing mathematical modelling in the secondary classroom. In: J. Watson and K. Beswick (eds.), *MERGA30* (Vol. 2, pp. 688–697). MERGA, Hobart.

E. Sudihartinih and T. Purniati (2018). Manipulative's of function translation. In *IOP Conference Series: Materials Science and Engineering*, 288(1), p. 012063. IOP.

P. Sullivan, M. Askew, J. Cheeseman, D. M. Clarke, A. Mornane, A. Roche, and N. Walker (2014). Supporting teachers in structuring mathematics lessons involving challenging tasks. *JMTE* 18(2), 1–18.

M. Swan (2015). *Using space efficiently: Packing a truck*. Mathematics assessment resource service. Shell Centre: https://www.map.mathshell.org/lessons.php

G. Williams (2014). Optimistic problem-solving activity: enacting confidence, persistence, and perseverance. *ZDM* 46(3), 407–422.

T. Wood (2002). What does it mean to teach mathematics differently? In: B. Barton, K. Irwin, M. Pfannkuch, and M. Thomas (eds.), *MERGA25*, (Vol. 1, pp. 61–67). MERGA, Auckland.

R. Zazkis, P. Liljedahl, and K. Gadowsky (2003). Conceptions of function translation: Obstacles, intuitions, and rerouting. *JMB* 22(4), 435–448.